WAVES 1

Waves

wave is a disturbance which propagates energy and momentum from one place to another without the transport of medium.

$\Rightarrow$ The medium should have elasticity, inertia and uniform density

Characteristics of Wave

$\Rightarrow$ The particles of the medium are executing simple harmonic motion.

$\Rightarrow$ The phase of vibration of the particle keep on changing.

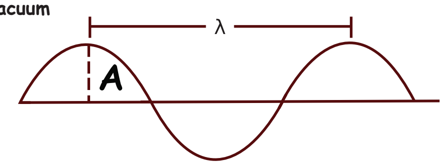
$\Rightarrow$ Wave carries energy and momentum.

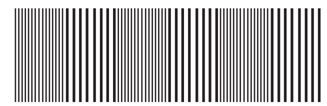
$\Rightarrow$ The velocity of the particle is not equal to velocity of wave.

Classification of waves

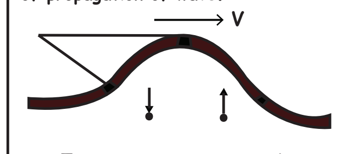
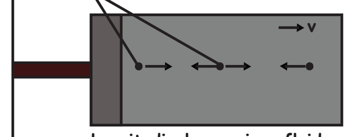
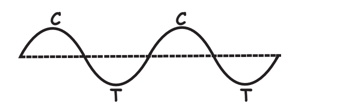
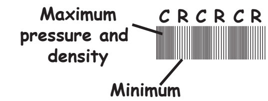
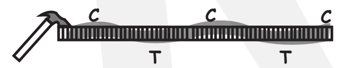
According to

Necessity of medium	Energy propagation	Dimension	Vibration of particles
Mechanical waves	Progressive waves	One Dimensional	Transverse waves
Non-mechanical waves(EMW) can travel in vacuum	Stationary waves	Two Dimensional	Longitudinal waves
		Three Dimensional	

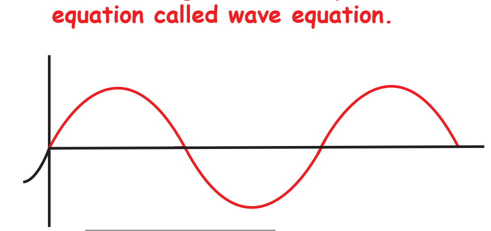




Classification of waves based on vibration of particles

Transverse waves	Longitudinal waves
Particles of the medium vibrates in a direction perpendicular to the direction of propagation of wave.  Transverse wave on a string	Particles of the medium vibrates in the direction of wave motion. Particles of the medium.  Longitudinal wave in a fluid
It travels in the form of crests (C) and troughs (T). 	It travels in the form of compression (C) and rarefaction (R). 
Transverse waves can be transmitted through solids, they can be setup on the surface of liquids. But they can not be transmitted through liquids and gases.  Transverse-wave in a rod	These waves can be transmitted through solids, liquids and gases because for these waves propagation, volume elasticity is necessary.  Longitudinal wave in a rod
Medium should posses the property of rigidity	Medium should posses the property of elasticity
Transverse waves can be polarised	Longitudinal waves can not be polarised
Movement of string of a sitar or violin, movement of the membrane of a tabla or Dholak, movement of kink on a rope, waves set-up on the surface of water.	Sound waves travel through air, vibration of air column in organ pipes vibration of air column above the surface of water in the tube of resonance apparatus.

All travelling wave satisfy a differential equation called wave equation.



$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2}$$

$$\frac{dy}{dt} = -\frac{\omega}{k} \times \frac{dy}{dx}$$

Velocity of particle = -Velocity wave × Slope of the graph

Equation of progressive wave

(i) $y = A\sin(\omega t - kx)$

(ii) $y = A\sin(\omega t - \frac{2\pi}{\lambda}x)$

(iii) $y = A\sin 2\pi\left[\frac{t}{T} - \frac{x}{\lambda}\right]$

(iv) $y = A\sin\frac{2\pi}{\lambda}(vt - x)$

(v) $y = A\sin\omega\left(t - \frac{x}{v}\right)$

The general equation of a plane progressive wave with initial phase is

$$\underbrace{y(x,t)}_{\text{Displacement}} = \underbrace{a}_{\text{Amplitude}} \sin(\underbrace{\omega t \pm kx \pm \phi_0}_{\text{Oscillating term phase}})$$

Angular frquency (ω), Initial phase (ϕ_0), Direction ($\pm$), Propagation (k)

Important Terms

A Amplitude ν Frequency

λ wave length T Time period

ω Angular frequency K Wave Constant

$\omega = \frac{2\pi}{T}$ or $2\pi\nu$ $K = \frac{2\pi}{\lambda}$

velocity of particle	Acceleration of particle
$v = \omega\sqrt{A^2 - y^2}$	$a = \omega^2 y$
$v_{max} = \omega A$	$a_{max} = \omega^2 A$

If v_{max} of the particle $= n \times v_{of\ wave}$

$\omega A = n \times \nu\lambda$ $n = \frac{2\pi A}{\lambda}$

$2\pi\nu A = n \times \nu\lambda$ $\lambda = \frac{2\pi A}{n}$

$A = \frac{n\lambda}{2\pi}$

Intensity of wave

$$I = \frac{\text{Energy}}{\text{time} \times \text{Area}} = 2\pi^2\nu^2 a^2 \rho v$$

$$I = \frac{P}{4\pi r^2} \qquad I \propto \nu^2 a^2$$

PHYSICS WALLAH

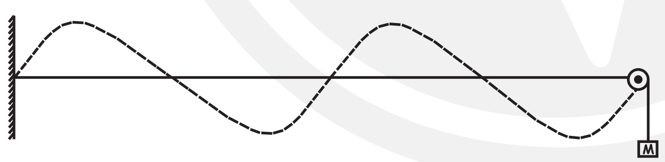
Rate of Energy Transmission

$$\frac{dK}{dt} = \frac{dU}{dt} = \frac{1}{4}\mu V\omega^2 A^2$$

Rate of an power Transmission

$$\frac{1}{2}\mu V\omega^2 A^2$$

Velocity of Transmission wave in a string.

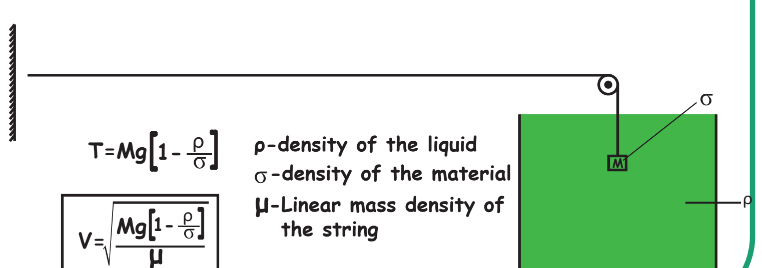


$V = \sqrt{\frac{T}{\mu}}$ $T = Mg$ M-Mass of block

$V = \sqrt{\frac{T}{\rho\pi r^2}}$ $\mu = \frac{m}{\ell}$ m-mass of string

$\frac{V_1}{V} = \sqrt{\frac{T_1}{T_2}} \times \frac{r_2}{r_1}$ $\mu = \rho A$ ρ-density of material of string

Case 1

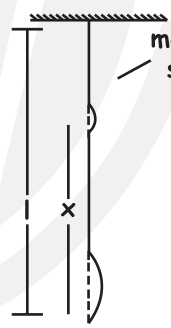


$T = Mg\left[1 - \frac{\rho}{\sigma}\right]$ ρ-density of the liquid

σ-density of the material

$V = \sqrt{\frac{Mg\left[1 - \frac{\rho}{\sigma}\right]}{\mu}}$ μ-Linear mass density of the string

Case 2



Velocity at any Point

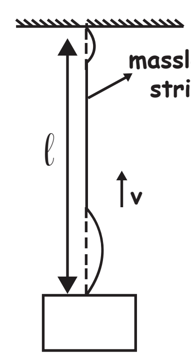
$$V = \sqrt{gx}$$

Time taken to reach the pulse at Top

$$T = 2\sqrt{\frac{\ell}{g}}$$

Case 3

Time taken to reach the pulse at top



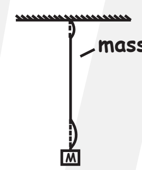
Velocity, $v = \sqrt{\frac{T}{\mu}}$

Time taken, $t = \frac{\ell}{\sqrt{\frac{T}{\mu}}} = \ell\sqrt{\frac{\mu}{T}}$

T= Tension in the string

μ= Linear mass density

Case 4



velocity at bottom

$$V_1 = \sqrt{\frac{Mg}{\mu}}$$

velocity at Top

$$V_2 = \sqrt{\frac{(M+m)g}{\mu}}$$

$$\frac{V_1}{V_2} = \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{M}{M+m}}$$

Velocity of Longitudinal Wave

$V = \sqrt{\frac{E}{\rho}}$ (E=Elasticity of the medium; ρ=density of the medium)

(1) As solids are most elastic while gases least i.e. $E_S > E_L > E_G$. So the velocity of sound is maximum in solids and minimum in gases

Velocity Sound (air) $\gamma = \frac{C_p}{C_v}$

$V = \sqrt{\frac{P}{\rho}}$ — Newton

$V = \sqrt{\frac{P\gamma}{\rho}}$ — Laplace

Mono atomic	γ= 5/3
diatomic	γ= 7/5

Factors affecting velocity of sound

→ Pressure	→ Density	→ Temp
· Velocity of sound in air is independent of pressure	$V \propto \frac{1}{\sqrt{\rho}}$	$V = \sqrt{\frac{RT\gamma}{m}}$
	$\frac{V_1}{V_2} = \sqrt{\frac{\rho_2}{\rho_1}}$	$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$

Temp Cofficient(α)

Increase in velocity of sound for 1°C or 1K rise in temperature of gas

Value of $\alpha = 0.608 \ \frac{m/s}{°C}$

$= 0.61$

Humidity

Humidity ↑

Speed of sound ↑

Sound travels faster in moist air than in dry air

Relation between Δx and ΔΦ

$$\Delta\Phi = \frac{2\pi}{\lambda}\Delta x$$

Reflection of Mechanical waves

Transverse wave
- Reflection from rigid end/denser medium → Phase change by π
- Reflection from free end/rarer medium → No phase change

Principle of superposition

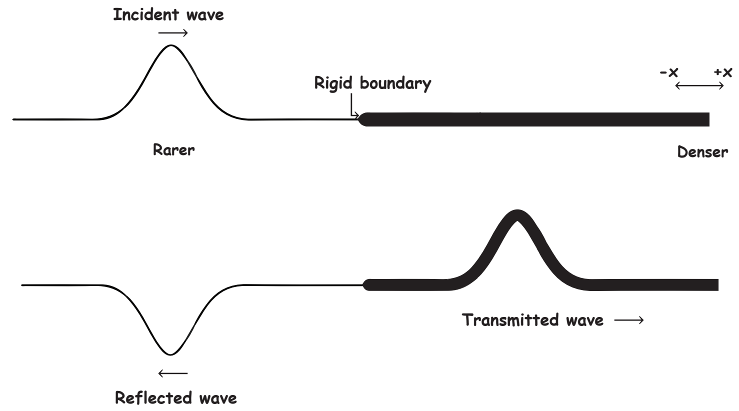
The displacement at any time due to number of waves meeting simulatoneously at a point in a medium is the vector sum of individual displacements due each one of the waves at that point of same time

Superposition

- Interference
  - Constructive
  - Destructive
- Beat
- Standing wave
  - In a string
  - In an open pipe
  - In a closed pipe

Wave combination of string

1) From rarer to denser medium



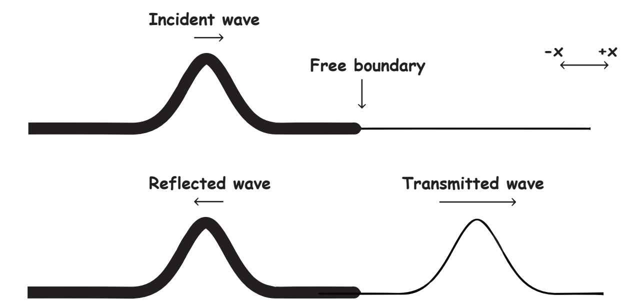
Incident wave $y_1 = a_1 \sin(\omega t - k_1 x)$

Reflected wave $y_r = a_r \sin(\omega t - k_1(-x) + \pi)$

$= -a_r \sin(\omega t + k_1 x)$

Transmitted wave $y_t = a_t \sin(\omega t - k_2 x)$

2) From rarer to denser medium



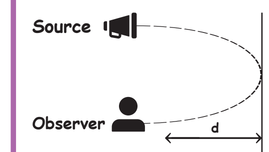
Incident wave $y_i = a_i \sin(\omega t - k_1 x)$

Reflected wave $y_r = a_r \sin(\omega t - k_1(-x) + 0)$

$= a_r \sin(\omega t + k_1 x)$

Transmitted wave $y_t = a_t \sin(\omega t - k_2 x)$

Echo



Source at distance "d" from screen

$t = \frac{d}{v} + \frac{d}{v} = \frac{2d}{v}$

Persistance of hearing for human ear is 0.1 sec

Conditions for echo:

if $t > 0.1 \Rightarrow \frac{2d}{V} > 0.1 \Rightarrow d > \frac{V}{20}$

Interference of sound wave

Condition:-

- Two waves of same frequency, same wavelength, same velocity
- Resultant intensities will be different from the sum of intensities of each wave seperately
- This is due to the interference of waves

$y_1 = a_1 \sin \omega t$, $y_2 = a_2 \sin(\omega t + \phi)$

ϕ-Phase difference between two waves

$\bar{y} = \bar{y}_1 + \bar{y}_2 \Rightarrow y = A \sin(\omega t + \theta)$

$A = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi}$

$\tan\theta = \frac{a_2 \sin\phi}{a_1 + a_2 \cos\phi}$

Intensity α A^2

$A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi$

$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\phi$

$\frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2$ $\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$

i) For Constructive interference:-

$\phi = 0, 2\pi, 4\pi, ---, 2\pi n$ when $n = 0, 1, 2, ---$

$x = 0, \lambda, 2\lambda, ---, n\lambda$, when $n = 0, 1, ---$

$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$

$= (\sqrt{I_1} + \sqrt{I_2})^2 = (A_1 + A_2)^2$

ii) For Destructive interference:-

when $\phi = \pi, 3\pi, 5\pi, --- (2n-1)\pi$; where $n = 1, 2, 3, ---$

$x = \frac{\lambda}{2}, \frac{3\lambda}{2}, ------(2n-1)\frac{\lambda}{2}$, where, $n = 1, 2, 3, -----$

$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$

$\Longrightarrow I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2 \propto (A_1 - A_2)^2$

Beats:-

- sound waves travelling in same medium with slightly different frequencies superimpose on each other.
- The intensity of resultant sound at particular position rises and falls regularly with time.
- The phenomenon of variation of intensity of sound with time at a particular position is called beats.

Point to remember:-

1) One beat:-

Maximum intensity (at t=0) → Becomes minimum intensity → Again acheives maximum

} One beat is formed

Beat period:-

Time interval between two sucessive beats (ie.two sucessive maximum of sound) is called beat period.

Beat frequency:-

No.of beats produced per second

Beat frequency:- $n = |n_1 - n_2|$

Beat period:- $T = \frac{1}{\text{Beat frequency}} = \frac{1}{|n_1 - n_2|}$

WAVES 2

Determination of Unknown Frequency

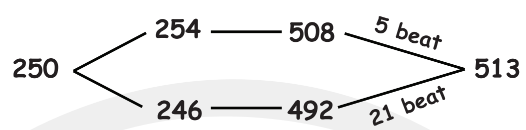
Let n_2 is the unknown frequency of tuning fork B, and this tuning fork B produce x beats per second with another tuning fork of known frequency n_1.
As number of beat/sec is equal to the difference in frequencies of two sources, therefore $n2 = n1 \pm x$

By loading	By filing
If B is loaded with wax so its frequency decreases	If B is filed, its frequency increases

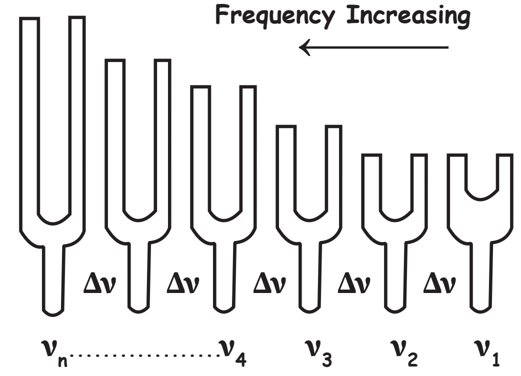
?) A Source of unknown frequency produces 4 beat/s when sounded with a source of Known frequency 250 Hz. The second harmonic of the source of unknown frequency gives 5 beat/s when Sounded with a source of frequency 513 Hz. The unknown frequency is?

a) 254 Hz   b) 246 Hz   c) 240 Hz   d) 260 Hz

Solution:

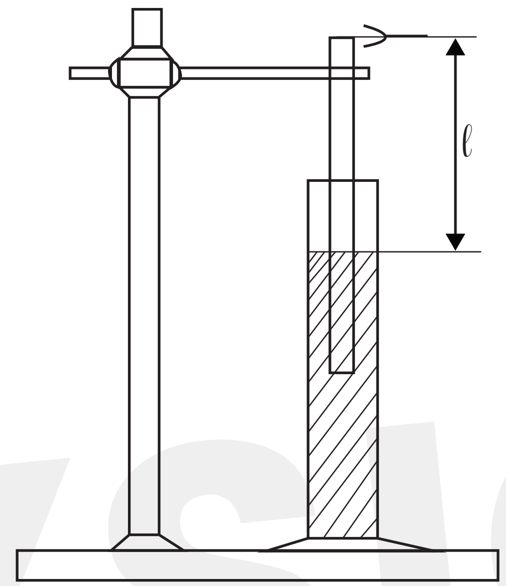


Hence unknown frequency is 254 Hz



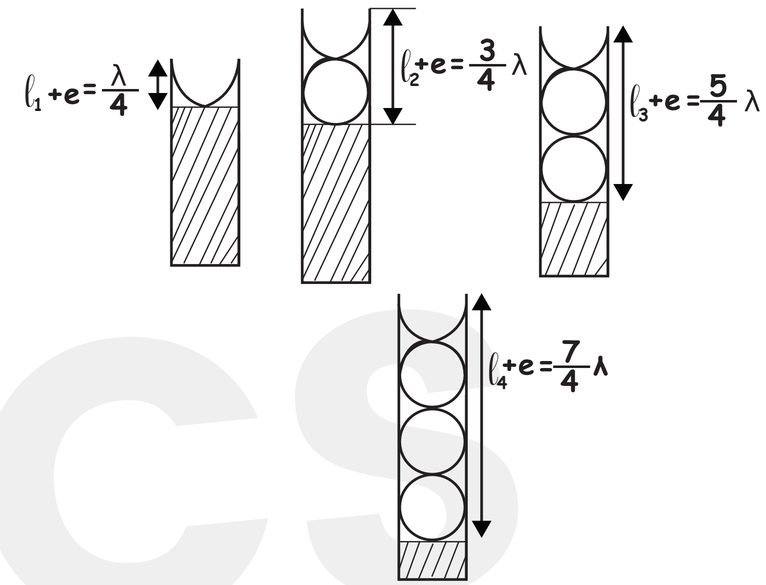
$\nu_{n^{th}} = \nu_1 + (n-1)\,\Delta\nu$

Resonance tube experiment



End correction:-

$e = \frac{1}{2}(\ell_2 - 3\ell_1)$



Standing Waves:

- When two progressive waves (both longitudinal and transverse) having same amplitude, time period, frequency moving along a straight line in opposite direction a superpose a new wave is formed. It is called stationary Or standing wave.

String

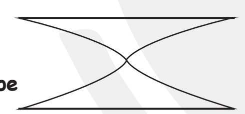
1:2:3

$y = 2a\sin(kx)\cos(\omega t)$

$\nu = \frac{n}{2\ell}\sqrt{\frac{T}{\mu}}$ $\ell = n \times \frac{\lambda}{2}$

$(n+1) \rightarrow$ node
$n \rightarrow$ antinode

Open pipe



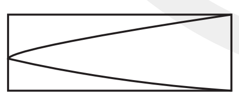
1:2:3

$y = 2a\cos(kx)\sin(\omega t)$

$\nu = \frac{n}{2\ell}\sqrt{\frac{\gamma P}{\rho}}$ $\ell = n \times \frac{\lambda}{2}$

$(n+1) \rightarrow$ antinode
$n \rightarrow$ node

Closed pipe

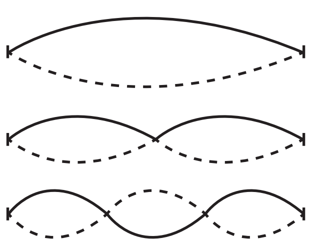


$y = 2a\sin(kx)\cos(\omega t)$

$\nu = \frac{n}{2\ell}\sqrt{\frac{\gamma P}{\rho}}$ $\ell = n \times \frac{\lambda}{4}$

n=1,3,5....

Note



Distance between two adjacent node & antinode is $\frac{\lambda}{2}$

Distance between two adjacent node & antinode is $\frac{\lambda}{4}$

Phase difference between 2 particle at both sides of node is 180° or π

Strain and pressure is maximum at node and minimum at antinode

Octave: The tone whose frequency is double the fundamental frequency is defined as Octave.

(i) If $n_2 = 2n_1$ it means n_2 is an octave higher than n_1 or n_1 is an octave lower than n_2.

(ii) If $n_2 = 2^3 n_1$ it means n_1 is 3-octave higher or n_1 is 3-octave lower.

(iii) Similarly if $n_2 = 2^n n_1$, it means n_2 is n-octave higher n_1 is n octave lower.

Unison: If the two frequencies are equal then vibrating bodies are said to be in unison.

Resonance: The phenomenon of making a body vibrate with it's natural frequency under the influence of another vibrating body with the same frequency is called resonance.

Comparative Study of Stretched Strings, Open Organ Pipe and Closed Organ Pipe

S . NO	Parameter	Stretched string	Open organ pipe	Closed organ pipe
1	Fundamental frequency or 1st harmonic	$n_1 = \frac{v}{2l}$	$n_1 = \frac{v}{2l}$	$n_1 = \frac{v}{4l}$
2	Frequency of or 2nd harmonic	$n_2 = 2n_1$ 1st overtone	$n_2 = 2n_1$ 1st overtone	Missing
3	Frequency of or 3rd harmonic	$n_3 = 3n_1$ 2nd overtone	$n_3 = 3n_1$ 2nd overtone	$n_3 = 3n_1$ 1st overtone
4	Frequency ratio of overtones	2:3:4.....	2:3:4.....	3:5:7.....
5	Frequency ratio of harmonics	1:2:3:4.....	1:2:3:4.....	1:3:5:7.....
6	Nature of waves	Transverse stationary	Longitudinal stationary	Longitudinal stationary

WAVES 3

Relation between loudness and intensity

$L \propto \log_{10}$ Intensity $I_0 = 10^{-12} W/m^2$

unit(dB)   unit W/m²

I_0=Threshold intensity

$dB = 10 \times \log_{10}\frac{I}{I_0}$

ΔL = change in loudness

ΔI = change in intensity

$L_1 = 10 \times \log_{10}\frac{I_1}{I_0}$ $I_1 \longrightarrow L_1$

$L_2 = 10 \times \log_{10}\frac{I_2}{I_0}$ $I_2 \longrightarrow L_2$

$L_2 - L_1 = 10\left[\log\left(\frac{I_2}{I_0}\right) - \log\left(\frac{I_1}{I_0}\right)\right]$

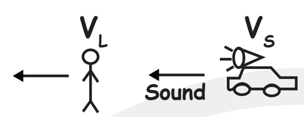
$\Delta L = 10\,\log\left(\frac{I_2}{I_1}\right)$

Doppler Effect

Whenever there is a relative motion between a source of sound and the listener, the apparent frequency of sound heard by the listener is different from the actual frequency of sound emitted by the source.

General equation
(when both source & listener are moving)

$$\nu' = \left(\frac{V \pm V_L}{V \pm V_S}\right)\nu$$



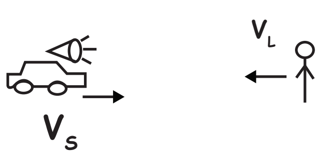
Case 1
(listener is stationary & source is approaching the listener)

$$\nu' = \left(\frac{V}{V - V_S}\right)\nu$$

Case 2
(The source is stationary & listener is approaching the source)

$$\nu' = \left(\frac{V + V_L}{V}\right)\nu$$

Case 3
(source & listener are approaching each other)



$$\nu' = \left(\frac{V + V_L}{V - V_s}\right)\nu$$

Case 4
(source is stationary, listener is moving away from the source)

$$\nu' = \left(\frac{V - V_L}{V}\right)\nu$$

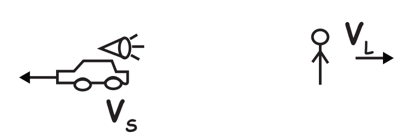
Case 5
(source is moving away from the listener, listener is stationary)

$$\nu' = \left(\frac{V}{V + V_S}\right)\nu$$

Case 6
(source and listener moving in same direction)

$$\nu' = \nu\,\frac{V + V_L}{V + V_s}$$

Case 7
(source and listener moving in opposite direction)

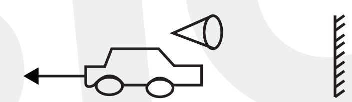


$$\nu' = \left(\frac{V - V_L}{V + V_s}\right)\nu$$

Case 8
(source approaching a stationary wall)

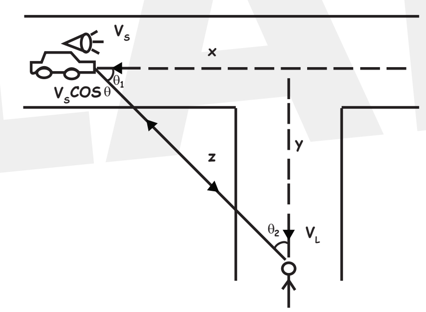
$$\nu' = \frac{V + V_s}{V - V_s}\,\nu$$

Case 9
(source is moving away from stationary wall)



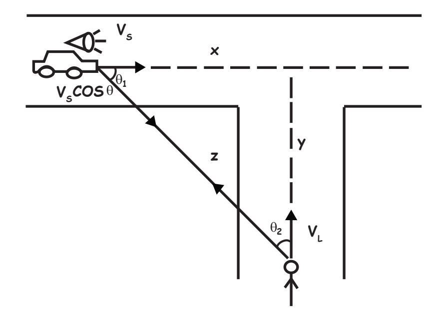
$$\nu' = \left(\frac{V - V_S}{V + V_S}\right)\nu$$

Case 10



$$\nu^1 = \left(\frac{V - V_L \cos\theta_2}{V + V_s \cos\theta_1}\right)\nu$$

Case 11

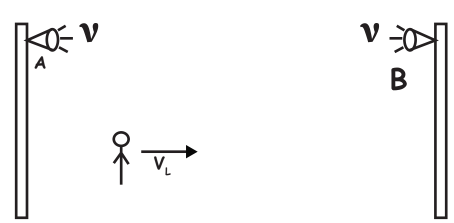


$$\nu' = \left(\frac{V + V_L \cos\theta_2}{V - V_s \cos\theta_1}\right)\nu$$

$$\cos\theta_1 = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\cos\theta_2 = \frac{y}{\sqrt{x^2 + y^2}}$$

Note:



$$\nu'_A = \left(\frac{V - V_L}{V}\right)\nu \qquad \nu'_B = \left(\frac{V + V_L}{V}\right)\nu$$

$$\text{Beat}(\Delta\nu) = \nu'_B - \nu'_A = \frac{\nu}{V}[V + V_L - V + V_L]$$

$$\Delta\nu = \frac{2V_L\nu}{V}$$